

Nullstellen berechnen

pq-Formel: $x^2 + bx + c = 0$

Definition: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$ $x^2 + px + q = 0$

Beispiele:

1. $f(x) = x^2 - 4x + 2$

Bedingung $f(x) = 0$

$$x^2 - 4x + 2 = 0$$

$$p = -4 \quad q = 2$$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} = -\frac{-4}{2} \pm \sqrt{\left(\frac{-4}{2}\right)^2 - 2}$$

$$= 2 \pm \sqrt{4 - 2} = 2 \pm \sqrt{2}$$

$$\Rightarrow \underline{x_1 = 2 + \sqrt{2}} \quad \underline{x_2 = 2 - \sqrt{2}}$$

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pq-Formel: $x^2 + bx + c = 0$

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Beispiele:

2. $f(x) = x^2 - x - 6$

Bedingung $f(x) = 0$

$$x^2 - x - 6 = 0$$

$$p = -1 \quad q = -6$$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} = -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 + 6}$$

$$= \frac{1}{2} \pm \sqrt{\frac{1}{4} + 6} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{24}{4}} = \frac{1}{2} \pm \sqrt{\frac{25}{4}} = \frac{1}{2} \pm \frac{5}{2}$$

$$\Rightarrow \underline{x_1 = \frac{6}{2} = 3} \quad \underline{x_2 = -\frac{4}{2} = -2}$$

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Mitternachtsformel: $ax^2 + bx + c = 0$ $a \neq 0$; $a > 0$

Definition: $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Beispiele:

1. $f(x) = 2x^2 - 4x - 6$

Bedingung $f(x) = 0$

$$2x^2 - 4x - 6 = 0$$

$$a = 2 \quad b = -4 \quad c = -6$$

$$\begin{aligned} x_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+4 \pm \sqrt{16 - 4 \cdot 2 \cdot (-6)}}{4} = \frac{4 \pm \sqrt{16 + 48}}{4} \\ &= \frac{4 \pm \sqrt{64}}{4} = \frac{4 \pm 8}{4} \Rightarrow \underline{x_1 = \frac{12}{4} = 3} \quad \underline{x_2 = \frac{-4}{4} = -1} \end{aligned}$$

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Mitternachtsformel: $ax^2 + bx + c = 0$

Definition:
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Beispiele:

1. $f(x) = 3x^2 + 5x - 2$

Bedingung $f(x) = 0$

$$3x^2 + 5x - 2 = 0$$

$$a = 3 \quad b = 5 \quad c = -2$$

$$\begin{aligned} x_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 4 \cdot 3 \cdot (-2)}}{2 \cdot 3} = \frac{-5 \pm \sqrt{25 + 24}}{6} \\ &= \frac{-5 \pm \sqrt{49}}{6} = \frac{-5 \pm 7}{6} \Rightarrow \underline{x_1 = \frac{2}{6} = \frac{1}{3}} \quad \underline{x_2 = \frac{-12}{6} = -2} \end{aligned}$$

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Direkter Vergleich:

Beispiel:

$$f(x) = 2x^2 - 4x - 6$$

$$\text{Bedingung } f(x) = 0$$

$$2x^2 - 4x - 6 = 0 \quad / :2$$

$$x^2 - 2x - 3 = 0$$

$$p = -2 \quad q = -3$$

$$x_{1/2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 + 3}$$

$$= 1 \pm \sqrt{1+3} = 1 \pm \sqrt{4} = 1 \pm 2$$

$$\Rightarrow \underline{x_1 = 1+2 = 3}$$

$$\underline{x_2 = 1-2 = -1}$$